Exercise 1. Cluster algorithms

Goal: Cluster algorithms can be used to reduce the critical slowing down substantially. This week you have to simulate either the 3D Ising model or the Potts model with $q = 2$ using a cluster algorithm. You may choose either the Swendsen-Wang or the Wolff algorithm.

Task 1: Find the connection probability in the Ising model.

Hint: In the Potts model the probability for connecting sites in the same state is $p = 1 - \exp(-\beta J)$.

Task 2: Implement either the Swendsen-Wang or the Wolff algorithm. Check your code by plotting the binder cumulant or the magnetic susceptibility around T_c.

Task 3 (OPTIONAL): Implement the other algorithm as well.

Task 4: Compare the performance of the algorithms and show that in the cluster algorithms the critical slowing down is substantially reduced. Measure the runtime and compute the linear autocorrelation time τ (relative to the observable E or M) of the cluster algorithm and compare it to the Metropolis algorithm. Report a table with τ and the Monte Carlo speed defined as

$$\mathrm{MC}_{\mathrm{speed}} = \frac{\mathrm{sweeps}}{\mathrm{time}} \cdot \frac{1}{\tau}$$

for at least three temperature values, say T_c, $T_c + 1$, $T_c - 1$, and fixed size. Make another table with the temperature fixed at T_c and varying lattice size. Interpret the results.

Hint: Remember that you can extrapolate τ from the autocorrelation series

$$\rho_{XX}(\Delta t) = \frac{\left\langle (X_t - \bar{X})(X_{t+\Delta t} - \bar{X}) \right\rangle_t}{\sigma^2}.$$